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Relationship Between Urban Scale and Traffic Congestion Based on the Scaling Laws

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Abstract: Studies on the relationship between urban scale and traffic congestion are mainly based on empirical study but ignore the mechanism behind it. This paper discusses the relationship between urban scale and traffic congestion using the scaling laws in complex system theory. The results show that the larger city can lead to more congestion and higher outputs. The use of scaling laws can enhance the understanding of the city and propose rational management measures, including that 1) city scale should be coordinated with traffic capacity; 2) the level of traffic congestion in cities with different scales cannot be compared directly; 3) people who enjoy convenience in big cities need to pay more traffic costs. The results show that the scaling laws can effectively explain the imbalance between traffic supply and demand and provide theoretical support for urban transportation planning and traffic management. **DOI:** 10.13813/j.cn11-5141/u.2019.0311-en

Keywords: urban transportation; traffic congestion; scaling laws

With the continuous progress of urbanization, traffic congestion has become one of the major urban issues, and limiting the scale of large cities has become one of the measures to alleviate this issue. However, this measure has caused widespread controversy. Literature [1] suggested that it was not useful to reduce congestion by limiting the scale of a city. Based on empirical analyses in China, Literatures [2–3] showed that the larger population can lead to the greater pressure on the transportation system, namely that the population has a positive correlation with traffic congestion. Thomson pointed out in his book *Urban Layout and Transport Planning* that city scale had an important impact on traffic congestion [4]. However, this study focused on empirical evidence and lacked theoretical research. This paper studies the relationship between urban scale and traffic congestion from a theoretical perspective by combining the scaling laws and empirical evidence.

1 The scaling laws

In 1638, Galileo put forward the following proposition in his book *Dialogues Concerning Two New Sciences* that everything in the world cannot generally be magnified in a simple linear scale. American physicist Geoffrey West proved this proposition theoretically and empirically and put forward the concept of the scaling laws in his book *Scale: The Universal Laws of Life and Death in Organisms, Cities and Companies*. Physicists have conducted further research and introduced a formula to study the scaling laws [5]: $Y = cX^k$. Given that c is a constant, this formula can be simplified as $Y \propto X^k$. Studies have shown that k as the power is very important and can even determine the properties of the whole system [5–6]. If $k = 1$, it is a linear relationship, namely that Y is doubled if X is doubled; if $k > 1$, it is a superlinear relationship; and if $k < 1$, it is a sublinear relationship.

The relationship between strength and weight of an organism is sublinear, because the strength is proportional to its area whereas the weight is proportional to its volume. According to geometry, area is proportional to the square of length, e.g., the area of a circle $S = \pi r^2$; and volume is proportional to the cube of length, e.g., the volume of a sphere $V = 4/3\ \pi r^3$. With a sphere with uniform density as an example, its weight is proportional to its volume, so it can be deduced that area (strength) is proportional to the $(2/3)^{\text{th}}$ power of volume (weight) and the formula is $S = \pi(3/4\pi)^{2/3}V^{2/3}$.

Chemist Lietzke verified Galileo's proposition with the weightlifting competition results of the Olympic Games in 1956 [4–5]. Taking the logarithm on both sides of $Y = cX^k$, we

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can obtain that Log$Y$ = Logc + kLog$X$. If X is weight and Y is strength, they can be plotted on double logarithmic coordinates (see Figure 1), which is basically a straight line with a slope of 0.675, close to the theoretical value of 2/3 ≈ 0.667. The above principles are also applicable to related studies on cities.

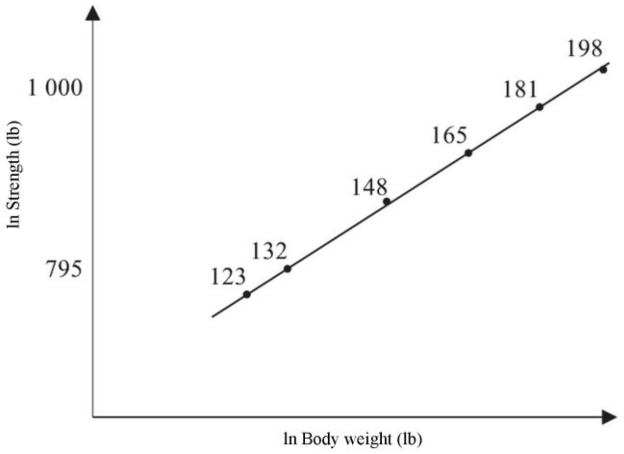


Figure 1 Weightlifter's weight and strength

Resource: Literature [5].

2 The larger city leading to a more economical situation

In the field of transportation, there is a rule that the larger ship can lead to a more economical situation, behind which is the scaling laws at work. English engineer Brunel realized that the capacity of a ship to carry cargo was determined by its volume, which is in proportion to the cube of its size. However, the drag force on a ship on water surface is proportional to the area of its bottom. Therefore, the resistance a ship must overcome is proportional to the $(2/3)^{th}$ power of the ship's load, which is a sublinear relationship. If the load is increased to 10 times, the power is only required to be 4.6 times of the original [5–6].

Similar to ships, it is also true for cities that the larger city indicates a more economical situation. Geoffrey West analyzed the relationship between the population and the number of gas stations in cities in France, Germany, the Netherlands and Spain. He found that more people lead to more gas stations, but the growth of the number of gas stations is slower than that of population: the number of gas stations is proportional to the 0.85^{th} power of the population. Therefore, this is a sublinear relationship, which means that the larger city (the larger population) leads to the fewer gas stations needed per capita [5–6].

Literature [5] revealed more research results. The total length of urban roads, water, electricity and the total length of gas pipelines are also proportional to the 0.85^{th} power of the population, namely urban infrastructure ∝ population$^{0.85}$. This result indicates that large cities are more efficient in infrastructure than small cities. This paper analyzes the relationship between urban population and road length in more than 600 cities in China and finds the power to be 0.87, which is close to the statistical results of overseas countries (see Figure 2).

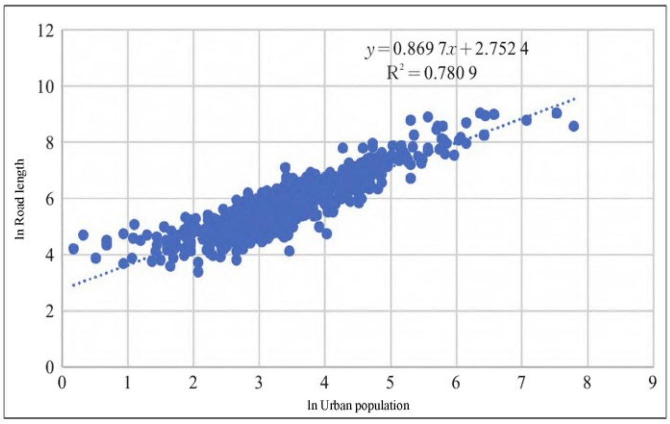


Figure 2 Relationship between urban population and road length in China's cities

Resource: 2016 China Urban Construction Statistical Yearbook

3 Larger city resulting in more congested situation

As mentioned above, the length of urban infrastructure is proportional to the 0.85^{th} power of the population. Divided by the population on both sides, it can be derived that the urban infrastructure/population ∝ Population$^{-0.15}$, namely that the larger urban population can lead to the fewer urban infrastructure per capita. In other words, the larger city, indicates the more efficient utilization of the facilities. In terms of traffic, when the growth exceeds the carrying capacity of the transportation system, traffic congestion will occur, which basically conforms to the traffic congestion mechanism in the world.

3.1 Empirical analysis

In this paper, the traffic congestion rankings of Chinese cities in the second quarter of 2018, released by the Baidu Inc., are selected as a reference (see Table 1) [7]. Table 1 shows that most of congested cities are large cities. In general, the peak congestion delay per capita is positively correlated with the population in the major cities of the United Kingdom (see Figure 3a) and that in the major metropolitan areas of the United States (see Figure 3b). In the United States, the congestion in New York and Chicago is less severe due to their developed public transportation systems: New York accounts for more than one third of the transit ridership in the United States.

Table 1 Congestion rank of the 2nd quarter released by the Baidu Inc.

City	Ongestion index for commute peak hours	Actual travel speed during commute peak hours/(km/h)	City	Ongestion index for commute peak hours	Actual travel speed during commute peak hours/(km/h)
Beijing	2.014	27.46	Nanjing	1.630	31.44
Harbin	1.919	25.25	Jinan	1.623	31.39
Shanghai	1.901	28.54	Mianyang	1.611	34.90
Changchun	1.893	28.68	Changsha	1.607	32.71
Chongqing	1.859	28.21	Tianjin	1.603	32.26
Guiyang	1.778	29.76	Zhuhai	1.590	33.02
Hohhot	1.719	29.96	Hangzhou	1.589	33.01
Shenyang	1.705	28.73	Nanning	1.584	31.32
Dalian	1.670	30.28	Dali Bai Autonomous Prefecture	1.579	29.22
Wuhan	1.657	29.80	Yibin	1.568	31.39
Kunming	1.654	32.95	Hefei	1.564	32.40
Leshan	1.642	29.38	Yangquan	1.563	29.11
Guangzhou	1.634	33.98			

Resource: Literature [7].

3.2 Mechanism analysis

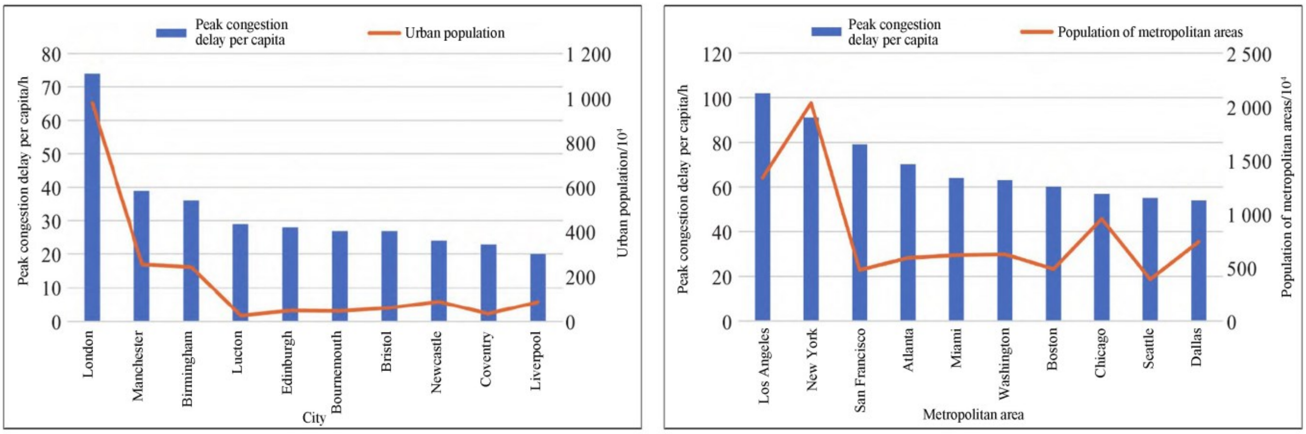
Why is there a sublinear relationship between urban infrastructure and population? Bettencourt [8] and Daganzo [9] performed mathematical reasoning and conducted theoretical analysis, respectively. This paper proposes a simplified analysis model based on the basic theory of the scaling laws.

1) Mathematical reasoning of Bettencourt

As for the origin of 0.85, Bettencourt of the Santa Fe Institute of the United States performed a mathematical reasoning and believed that it should be close to 5/6 [8]. The reasoning process is as follows.

The basic budget per capita in a city is proportional to the urban population density. N is set as the total urban population and A as the urban area. Then the urban population density is $n = N/A$, and the basic budget per capita is proportional to N/A.

On the other hand, the cost per capita is proportional to the $(1/2)^{th}$ power of A. In order to survive, a city must have the cost equal to the basic budget, so A to the power of 1/2 is proportional to N/A, i.e., $A \propto N^{2/3}$. Therefore, the area of an ideal city is proportional to the $(2/3)^{th}$ power of its total population, which suggests that larger cities are more congested, but on the other hand, larger cities would save more land.



a Cities in the United Kingdom (2017)

b Metropolitan area in the United States (2017)

Figure 3 Congestion rank and population in the United Kingdom and the United States

Resource: INRIX 2017 Traffic Scorecard.

In terms of infrastructure, the average distance between two people in a city d would be equal to the square root of the area per capita, namely $d \propto \sqrt{A/N}$. For a city with good infrastructure, the length of urban roads per capita should be equal to the average distance between two people. Therefore, the total length of urban roads Nd should be proportional to $\sqrt{AN}$. Considering that A is proportional to the $(2/3)^{th}$ power of N, it can be derived that $Nd \propto N^{5/6}$, where 5/6 = 0.833, which is the origin of the power of 0.85 mentioned above.

2) Theoretical analysis of Daganzo

In October 2018, Daganzo, an American Academy of Engineering academician, gave a more straightforward explanation in a speech [9]. In a Central Business District (CBD) of a city, the traffic demand is proportional to the CBD's area, which is proportional to the square of its side length. In addition, the number of vehicles that can pass through the CBD boundary is proportional to the CBD's perimeter, which is proportional to the first power of its side length. When the side length becomes larger, namely that the scale of CBD increases, the contradiction between traffic demand and traffic supply becomes more prominent (see Figure 4).

If the cumulative effect of time is taken into account, this contradiction will be further intensified. The amount of traffic that can be accommodated by city streets is proportional to the area of the city, but the amount of traffic that needs to coexist is proportional to the product of the area and the

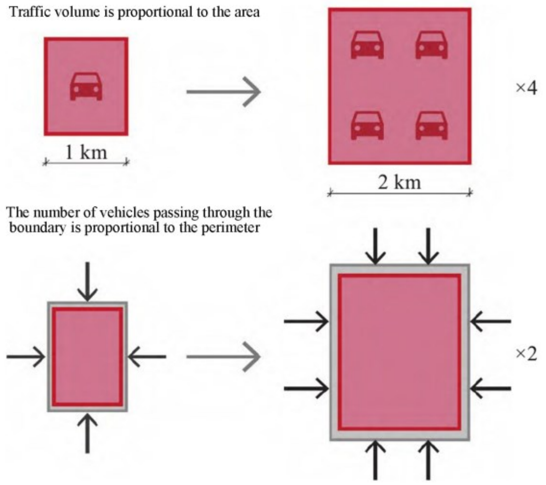


Figure 4 Nature of the contradiction between supply and demand: the scale dilemma

Resource: Literature [9].

diameter of the city. Therefore, the larger city may result in the more concentrated traffic peaks (see Figure 5). This rule is reflected in the fact that the duration of the traffic peak in large cities is generally longer than that in small cities.

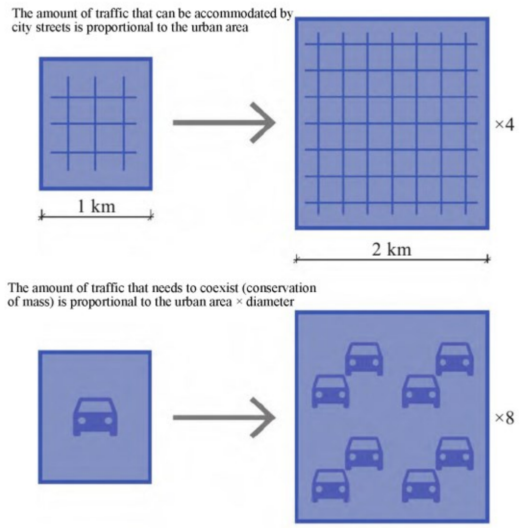


Figure 5 Scale dilemma considering time indicator

Resource: Literature [9].

3) Simplified analytical model

Assuming that a city is composed of $m \times m$ grid cells (m is odd, see Figure 6) and each grid cell generates an equal amount of travel volume T to other cells, then the outbound travel volume of the central grid cell can be formulated as $C = (m^2 - 1)*T/2$. This formula shows that the travel volume is a quadratic function of the city scale and this function is superlinear. With a 5 × 5 grid city and a 7 × 7 grid city as an example, the city scale increases by 1.4 times, but the outbound travel volume of the central grid cell increases by 2 times.

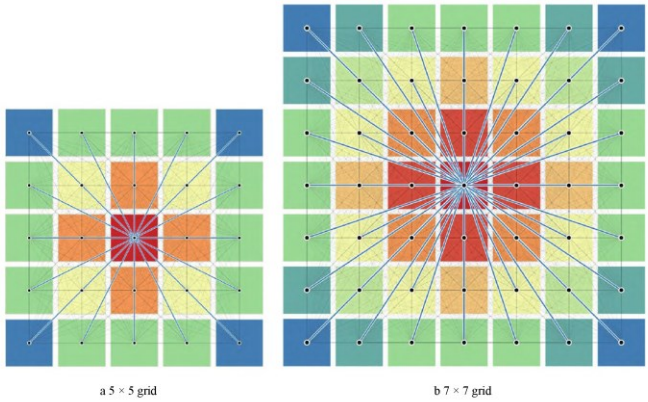


Figure 6 Travel demand between city centers and outside groups in cities with different scales

The formula of the city center's outbound travel volume and the city scale shows that the larger city scale can lead to the larger increase in the city center's outbound travel volume caused by a unit increase in the city scale. In other words, the city center's outbound travel volume grows much faster than the area of the city (see Figure 7). Therefore, the intensity of additional trips caused by urban expansion in large cities is higher than that in small cities.

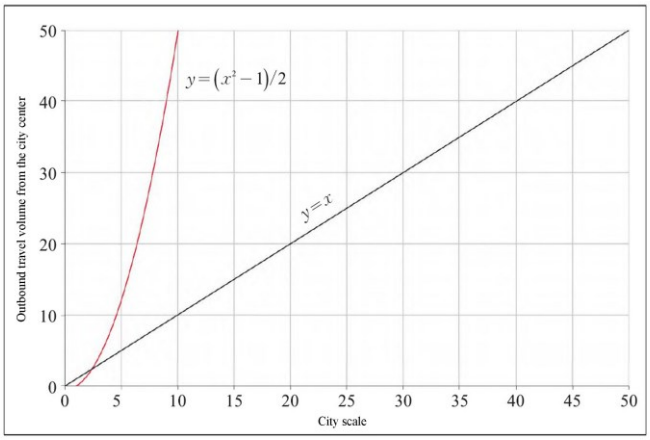


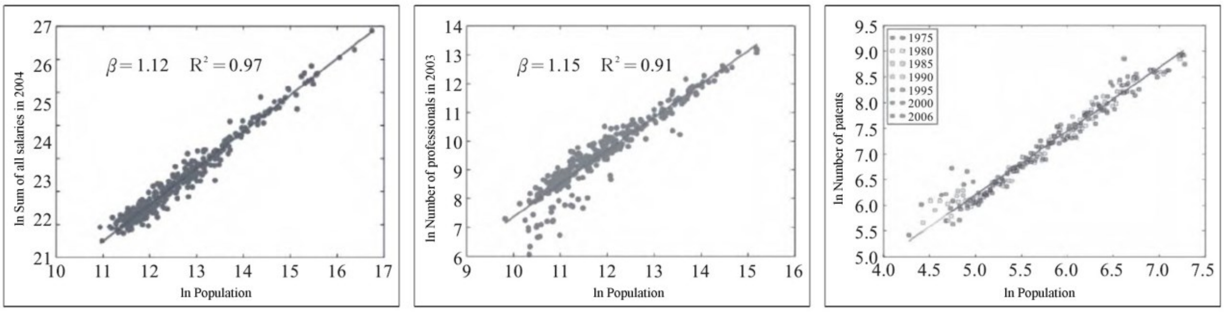
Figure 7 Relationship between city scale and travel demand between city centers and outside groups

4 The larger city indicating higher output

The above analysis shows that the larger city can result in the more congested traffic. Should the development of large cities be controlled? Studies on the scaling laws show that the answer is "no", because the output and the size of a city have a superlinear relationship, namely that the larger city indicates the higher output [4–5].

4.1 Empirical research

Geoffrey West analyzed the relationship of aggregate economic output, number of professionals and number of patents with population of various cities in the United States. He found that these indicators and urban scale also conform to the scaling laws (see Figure 8) with the power of about 1.15. The formula is urban output = population$^{1.15}$. It reflects a superlinear relationship, which means that urban output grows faster than the expansion of urban scale. Considering a large city with the population of 10 million and a small city with the population of 100 000, the population of the former is 100 times of the latter. Since 100 to the power of 0.85 equals 50, and 100 to the power of 1.15 equals 200, when a city's population increases by 100 times, its infrastructure only needs to increase by 50 times, but its output will increase by 200 times. From the perspective of the input–output ratio, the larger city indicates the better result.



a Sum of all salaries b Number of professionals c Number of patents

Figure 8 Relationship between urban indicators and city scale in cities of the United States

Resource: Literature [5].

Literatures [10–11] also showed that the economic growth brought by urban population agglomeration in China conformed to the scaling laws, and the power coefficient was higher than that of the United States. When urban population doubles, the economy grows by 122% (see Figure 9) in China, whereas it grows by 111% in the United States.

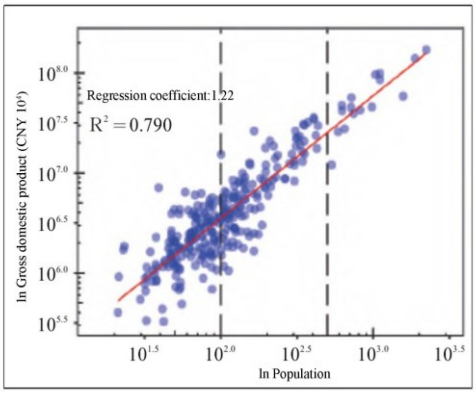


Figure 9 Scaling laws of urban population and gross domestic product in China

Resource: Literature [7].

4.2 Mechanism analysis

Why is the relationship between the urban output and the population superlinear? The reason is that the urban output comes from the connections between people. The urban output is not determined by the number of people but by the number of connections. The number of connections between n people is $D=C_n^2=\frac{n!}{2!(n-2)!}=\frac{n(n-1)}{2}$, namely that D is a quadratic function of n. For example, there are 10 connections among five people but 15 connections among six people. The number of connections grows much faster than the population (the actual total number of connections between people in a city is not a quadratic function of the population due to various constraints, but it is still a superlinear function). At the same time, the greater population density indicates the more connections each person can easily access and the greater total number of connections in the city.

5 Enlightenment from the relationship between urban scale and traffic congestion

1) Urban scale should be coordinated with traffic capacity.

Trees cannot be infinitely tall because they are bound by the scaling laws. The volume and weight of a tree are proportional to the cube of its size, and its supporting strength is determined by its cross-sectional area, which is proportional to the square of its size. If the height of a tree increases by 10 times, its volume and weight will increase by 1000 times, but its supporting force will only become 100 times of the original. Hence the tree needs to bear 1 000 times of the original weight with 100 times of the original strength. If the tree keeps growing, the tree will not be able to bear its own weight sooner or later.

The same problem exists in urban scale. Without considering the capacity of infrastructure, the larger city indicates the better result. However, the traffic capacity of a city is limited, inevitably leading to more traffic congestion when its urban scale becomes larger. Although some cities have partially alleviated traffic congestion by coordinating land use with transportation and strengthening traffic management, in general the larger city can lead to the more congested traffic. The urban scale should be controlled when traffic congestion starts to affect the normal operation of a city. For example, Beijing proposed to distribute the non-core functions of the capital, which is necessary from the perspective of alleviating traffic congestion.

Urban scale is also related to the level of transportation technology. The radius of a city is roughly equal to the distance that residents can travel within 1 hour. In the walking era, the city radius was about 4 km. It can reach 8 km–10 km in the bus era and exceed 25 km in the era of rail transit and cars.

2) The level of traffic congestion in cities with different scales cannot be compared directly.

The larger city may result in the more congested traffic. It is not appropriate to directly compare congestion indexes of cities with different scales. Instead, they should be compared after stratification or after they are converted based on the scaling laws. Based on this method, traffic congestion comparisons and traffic congestion mitigation plans can better conform to the city's own development law.

3) People who enjoy economic convenience in large cities need to pay more traffic costs.

Larger urban scale can bring more economic convenience and more efficient output to a city, but the city has to face the fact that the traffic is more congested. Cities can alleviate traffic congestion through various measures, but large cities are bound to pay more traffic costs. Literature [12] pointed out that some overseas scholars believed that congestion was a concomitant phenomenon of economic development, and the only way to cure congestion is economic recession. Under the premise of constant economic growth, the supply of new transportation facilities will attract new pass-through demand and attract new urban function agglomeration, which will not reduce the congestion but aggravate it.

6 Conclusion

According to the scaling laws, the larger city indicates the better results, and cities always tend to grow larger. First, the larger city can lead to the more intensive infrastructure. Second, the larger city can result in the higher output. However, when traffic demand exceeds traffic capacity, traffic congestion will occur, and the larger city indicates the more congested traffic. The objective law that trees cannot grow indefinitely implies that cities cannot bear to grow indefinitely and city scale should be coordinated with traffic capacity. This paper uses the scaling laws in the complex system theory to study the essence of traffic congestion. The relevant basic theories need to be further improved, and more data needs to be collected to verify the mechanism of traffic congestion revealed in this paper. Many factors could cause urban traffic congestion, such as urban space structure and transportation mode composition, which need to be studied further.

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